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Monday, May 12th, 1856.

JAMES HENTHORN TODD, D.D., President,
in the Chair.

Thomas H. Ledwich, Esq., and John H. Otway, Esq., were elected Members of the Academy.

The Rev. John H. Jellett read a Paper, by Mr. Thomas J. Campbell, on the solution of cubic equations.

"To resolve the cubic equation,

$$x^3 + ax^2 + bx + c = 0,$$

put $x = x' + z$, and the equation becomes

$$x'^3 + (3z + a)\,x'^2 + (3z^2 + 2az + b) \times x\,(z^3 + az^2 + bz + c) = 0,$$

which may be proved by development, for

$$\begin{array}{rl} x^3 = & x'^3 + 3zx'^2 + 3z^2x' + z'^3 \\ ax^2 = & \qquad ax'^2 + 2azx' + az^2 \\ bz = & \qquad\qquad\quad bx' + bz \\ c = & \qquad\qquad\qquad\quad c \end{array}$$

$$\therefore x^3 + ax^2 + bz + c = x'^3 + (3z + a)x'^2 + (3z^2 + 2az + b)x + z^3 + az^2 + bz + c.$$

Call the member on the right and left of this equation fx and $f'x$ respectively:

$$\therefore f'x = x'^3 + (3z + a)\,x'^2 + (3z^2 + 2az + b)\,x' + (z^3 + az^2 + bz + c) = 0.$$

My object is to reduce $f'x$ to the form of

$$x''^3 + Ax''^2 + \tfrac{1}{3}A^2x' + c' = 0,$$

where $x'' = f''x$ (or another function of $f'x$), and thus to find x'' by completing the cube, for a similar reason as we complete the square in equations of the second degree.

"But to effect this important relation of the coefficients of $f'x$, so as to reduce it to the form I have mentioned, we will divide both sides by

$$(z^3 + az^2 + bz + c)\, x'^3,$$

and thus we will have a new form of the equation in these terms, viz. :—

$$x'^{-3} + \left(\frac{3z^2 + 2az + b}{z^3 + az^2 + bz + c}\right)x'^{-2} + \left(\frac{3z + a}{z^3 + az^2 + bz + c}\right)x'^{-1}$$
$$+ \frac{1}{(z^3 + az^2 + bz + c)} = 0,$$

which is an equation the roots of which are the reciprocals of those of $f'x$. Now, let me compare the model equation

$$x''^3 + Ax^2 + \tfrac{1}{3}A^2x' + c$$

to the above, and we see that

$$A = \frac{3z^2 + 2az + b}{z^3 + az^2 + bz + c};$$

also

$$\tfrac{1}{3}A^2 = \frac{3z + a}{z^3 + az^2 + bz + c};$$

or

$$A^2 = 3\left(\frac{3z + a}{z^3 + az^2 + bz + c}\right) \therefore \left(\frac{3z^2 + 2az + b}{z^3 + az^2 + bz + c}\right)^2 = 3\left(\frac{3z + a}{z^3 + az^2 + bz + c}\right);$$

or, by development and clearing of fractions, we get the following function of z, viz. :—

$$(3z^2 + 2az + b)^2 = 3\,(3z + a)(z^3 + az^2 + bz + c)$$

$$\left\{\begin{matrix} 9z^4 + 12az^3 + (6b + 4a^2)z^2 \\ + 4abz + b^2 \end{matrix}\right\} = \left\{\begin{matrix} 9z^4 + 12az^3 + (9b + 3a^2)z^2 \\ + (9c + 3ab)z + 3ac \end{matrix}\right\}.$$

Hence, by contraction or transposition, we get the following quadratic :—

$$(3b - a^2)\, z^2 + (9c - ab)\, z + (3ac - b^2) = 0.$$

Hence

$$z = \frac{-(3b - a^2) \pm \sqrt{\{(3b - a^2)^2 - 4(3ac - b^2)(3b - a^2)\}}}{2(3b - a^2)},$$

but in the equation

$$x''^3 + Ax''^2 + \tfrac{1}{3}A^2x'' + c' = 0, \quad x'' = -\tfrac{1}{3}A + \tfrac{1}{3}(A^3 - 27c')^{\frac{1}{3}},$$

by completing the cube, and transposing c. Therefore, also,

$$x'^{-1} \text{ or } \frac{1}{x'}$$

$$= -\tfrac{1}{3}\,\frac{3z^2 + 2az + b}{z^3 + az^2 + bz + c} + \tfrac{1}{3}\left\{\left(\frac{3z^2 + 2az + b}{z^3 + az^2 + bz + c}\right)^3 - \frac{27}{z^3 + az + bz + c}\right\}^{\frac{1}{3}}$$

$$\therefore x'$$

$$= 3\left\{-\frac{3z^2 + 2az + b}{z^3 + az^2 + bz + c} + \sqrt[3]{\left(\frac{3z^2 + 2az + b}{z^3 + az^2 + bz + c}\right)^3 - \frac{27}{z^3 + az^2 + bz + c}}\right\}^{-1};$$

but $x' + z = x$, by the hypothesis in the original equation:

$$\therefore x = z$$

$$+ 3\left\{\frac{(3z^2 + 2az + b)}{z^3 + az^2 + bz + c} + 3\left(\frac{3z^2 + 2az + b}{z^3 + az^2 + bz + c}\right)^3 - \frac{27}{z^3 + az^2 + bz + c}\right\}^{-1};$$

consequently, substitute the foregoing value of z into this last formula, and it gives the value of the unknown quantity in any cubic of the form that I proposed."

Rev. Professor Haughton made a communication "On the Depth of the Sea deducible from Tidal Observations," of which the following is an abstract.

He stated that, in consequence of his having succeeded in separating the effects of the sun and moon in the diurnal tide, he was enabled to make calculations of the depth of the sea in which the tidal wave was produced, which he believed to be worthy of the greatest attention. The depth of the sea may be inferred from three distinct observations, viz., of heights of diurnal tide; of solitidal and lunitidal intervals; and of the age of the lunar tide compared with the lunitidal interval.